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Robust Optimization in Discrete Design Space for Constrained Problems

Kwon-Hee Lee*
Dong-A University, Busan 604-714, Republic of Korea
and
Gyung-Jin Park†
Hanyang University, Ansan 425-791, Republic of Korea

Robust design in discrete design space is defined as a discrete design that is insensitive to external uncertainties or variations. The application of robust discrete design is not prevalent yet due to high computational cost. A relatively simple method is proposed to select discrete and robust optimum. At first, the discrete design is achieved as the postprocess of conventional optimization. An orthogonal array is established around a conventional optimum, and the characteristic functions are evaluated. The characteristic function is defined by considering the robustness of the objective and constraints. The parameter design of the Taguchi method is introduced to obtain the robust solution in discrete space. The present method has insensitive performance to variations of the design variables within the selected discrete values enhancing the feasibility of constraints. To enhance feasibility, ranking the estimators of the characteristic function is developed. Several structural problems are solved to show the usefulness of the present method.

I. Introduction

ENGINEERING optimization technology has been exploited extensively as an automatic design tool for the design of structures.[1,2] Structural optimization is to discover a design with the highest performance satisfying imposed design criteria. The structural designs are determined in a discrete design space when the members are forced to be selected from existing or standardized products. Although continuous optimization delivers an excellent solution, the result should be modified to have discrete values for practical applications. In many practical designs, rounded-up (one-step higher) values are taken without giving further consideration. When the rounded-up values are chosen, the stress and displacement constraints are usually satisfied due to the excessive design. However, the eigenvalue constraints are not guaranteed. Therefore, a method is needed to overcome the difficulties.

Various methods have been suggested for discrete optimization.[3] They are approaches using the interactive optimization process,[4] branch and bound method,[5] dual method,[6,7] etc. Several algorithms such as the genetic algorithm and tabu search are used for the discrete design.[8–10] The number of function calculations is significantly large with these methods. In structural optimization, a function calculation is a finite element analysis, which could be very expensive for large-scale structures, limiting its use for practical application.

Also, a new design trend has emerged to consider the robustness. From the viewpoint of optimization, the robustness of the objective function makes the system performance insensitive to uncertainties. Rather, the robustness of the constraint function is defined by the feasibility condition that indicates that the optimum considering the uncertainties always lies in the feasible region. In the suggested robust design, the uncertainties are limited to the variations of design variables, whereas the rest are treated as constants.

The robust optimization methods in the continuous design space have been developed by several researchers.[11–16] In Refs. 17 and 18, the authors proposed robust optimization by using sensitivity information. However, studies for robust optimization are not well advanced in the discrete design space because of the difficulties in dealing with constraint feasibility. In this research, a method using the robust design of the Taguchi philosophy[19] (also see Ref. 20) is developed to perform the discrete and robust design. A discrete design using Taguchi's parameter design concept[19] for an unconstrained problem is reviewed in the following section (see Ref. 21). It is expanded to constrained problems by defining an appropriate characteristic function. A characteristic function with the standard deviation and the penalty function has been defined to consider the robustness. The standard deviation is relevant to the robustness of the objective function, whereas the penalty function composed of Lagrange multipliers, maximum violation, and scale factor controls the constraint robustness.

The method has been applied to the postprocess of constrained optimization. After the constrained optimization is performed in the continuous design space, an orthogonal array, called an inner array, with discrete values, is established around the continuous optimization results.[19–21] For each row of the inner array, an orthogonal array, called an outer array, with variations of design variables, is established, and the characteristic function is calculated. The outer array is adopted in the numerical experiments to include the effect of multiple experiments. The process obeys the parameter design scheme except that the signal-to-noise ratio (SNR)[19,20] is replaced by the characteristic function. However, the optimum evaluated through the analysis of the characteristic function does not guarantee feasibility. Thus, the characteristic function estimators with respect to all combinations are ranked by ascending order. The estimator is a linearly approximated value. Therefore, the characteristic function is calculated according to the prescribed order made by the estimator. The characteristic function is made by the outer array, and the feasibility is checked. The combination with the smallest estimator satisfying the constraints is selected as an optimum.

Various example problems are solved. They are well-known standard problems, which include the three-bar truss and the one-bay, two-story frame.[1,2] As a practical application, the design of a space frame in an electrical vehicle is carried out.[11] An optimization software IDESIGN3.3 (Ref. 22) is used for the optimization process and a module for the discrete design has been attached to the software.

II. Robust Design for Unconstrained Optimization Problems

Unconstrained optimization finds design variables while minimizing objective functions without any constraints. This problem can be relegated to the smaller-the-better-type characteristic

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*Assistant Professor, Mechanical Engineering, 840 Hadan-2-Dong.
†Professor, Mechanical Engineering, 396 Daehak-Dong, Kyunggi-do; gjpark@hanyang.ac.kr. Member AIAA.

problems in the Taguchi[19] method (see Ref. 21). In the case of the smaller-the-better-typeproblems, the SNR is defined as

$$\mathrm{SNR}_s = -10\log_{10}\left[\frac{1}{N_{\mathrm{out}}}\sum_{i=1}^{N_{\mathrm{out}}} f_i(\boldsymbol{x})^2\right] \tag{1}$$

where $f_i(\boldsymbol{x})$ is the ith characteristic or objective function, $\boldsymbol{x}$ are the vectors for design variables, and N_{out} is the number of function calculations considering the uncertainties.[19,20] In this research, the uncertainties are confined to the variations of the design variables. Equation (1) is decomposed into the mean μ_f and the standard deviation s_f of the characteristics as

$$\mathrm{SNR}_s = -10\log_{10}\left[\mu_f^2 + \frac{N_{\mathrm{out}}-1}{N_{\mathrm{out}}}s_f^2\right] \tag{2}$$

where

$$\mu_f = \sum_{i=1}^{N_{\mathrm{out}}} f_i(\boldsymbol{x}), \qquad s_f^2 = \sum_{i=1}^{N_{\mathrm{out}}} \frac{[f_i(\boldsymbol{x}) - \mu_f]^2}{N_{\mathrm{out}}-1}$$

From SNR_s of Eq. (2), it is found that SNR_s is confounded with the effects on the mean and the standard deviation of the characteristics.[23] From the viewpoint of robust design, it is an advantage to include the effect on the standard deviation only in Eq. (2) because the suggested approach is developed for the postprocess of conventional optimization.

For an unconstrained problem, a multiobjective function modifying the SNR_s of Eq. (2) is represented as

$$\Phi(\boldsymbol{x}) = \alpha\cdot\left(\mu_f/\mu_f^*\right) + (1-\alpha)\cdot\left(s_f/s_f^*\right) \tag{3}$$

where α is the weighting factor and μ_f^* and s_f^* are the function values at the optimum considering only the mean and the standard deviation as an objective function, respectively. The values μ_f and s_f are the functions of the design variable vector $\boldsymbol{x}$. The value of the weighting factor α is determined by the importance of minimization and robustness. The discrete values of the design variables are selected around the conventional optimum. The minimization of the objective function is somewhat achieved by conventional optimization. Therefore, by only considering the robustness of the objective function, the multiobjective function of Eq. (3) can be reduced as

$$\Phi(\boldsymbol{x}) = s_f \tag{4}$$

The mean and the standard deviation of the objective function in Eqs. (3) and (4) are evaluated from the outer arrays in which the uncertainties such as the tolerances of the design variables are included. Actually, the standard deviation implies the magnitude of interval sensitivity with respect to the variations on design variables. The way to reach the optimum levels is the same as that in the parameter design of the Taguchi method.[19]

III. Robust Design for Constrained Problems

A. Characteristic Function

A characteristic $f(\boldsymbol{x}) + P(\boldsymbol{x})$ can be regarded as the new objective function when $P(\boldsymbol{x})$ is the penalty function obtained from the constraint violation. If the scaling is not considered in Eq. (3), a multiobjective function with the new objective function is expressed as

$$\Phi(\boldsymbol{x}) = \alpha\cdot\mu_{f+P} + (1-\alpha)\cdot s_{f+P} \tag{5}$$

where μ_{f+P} is the sample mean and s_{f+P} is the sample standard deviation of the new objective function, respectively. If $f(\boldsymbol{x})$ and $P(\boldsymbol{x})$ are independent, $\mu_{f+P} = \mu_f + \mu_P$ and $s_{f+p}^2 = s_f^2 + s_p^2$. Equation (5) then is as follows:

$$\Phi(\boldsymbol{x}) = \alpha\cdot(\mu_f + \mu_P) + (1-\alpha)\cdot\sqrt{s_f^2 + s_P^2} \tag{6}$$

In the same manner, the robust design in the discrete space is developed as the postprocess of conventional optimization for the constrained problems. On the right-hand side of Eq. (6), only μ_P of the first term and only s_f^2 of the second term are considered for finding an optimum. In the first term of the right-hand side, μ_f is not included because minimization is achieved by conventional optimization, whereas s_P^2 in the second term is not included because the standard deviation of the penalty function is meaningless. Thus, Eq. (6) is reduced to

$$\Phi(\boldsymbol{x}) = \alpha\cdot\mu_P + (1-\alpha)\cdot s_f \tag{7}$$

To make the penalty function represented as the first term of Eq. (7) more conservative, the penalty function $P(\boldsymbol{x})$ is defined with the Lagrange multipliers, constraint violation, and scaling factor. Thus, the first term of Eq. (7) is replaced by Eq. (8) as follows:

$$P(\boldsymbol{x}) = \sum_{j=1}^{m} \lambda_j \times \max[0, v] \times z \tag{8}$$

where m is the number of constraints, λ_j is the Lagrange multiplier of the jth constraint, v is the maximum violation of the constraints, and z is a scale factor. The penalty function includes the Lagrange multiplier. The optimum sensitivity theorem is as follows[2]:

$$g_j[\boldsymbol{x}(e_j)] \le e_j, \qquad j = 1, \ldots, m \tag{9}$$

$$\frac{\partial f(\boldsymbol{x}^*)}{\partial e_j} = \lambda_j \tag{10}$$

where g_j is the jth constraint function, e_j is a small value, and $f(\boldsymbol{x}^*)$ is the objective function at the optimum $\boldsymbol{x}^*$. From Eqs. (9) and (10), it is shown that the larger Lagrange multiplier has a larger influence on the optimum when a small variation is considered. Thus, the design with the larger Lagrange multiplier can easily become infeasible when the tolerances of design variables are considered. The penalty function $P(\boldsymbol{x})$ has the same scale with the objective function. With Eq. (8), Eq. (7) is rewritten as follows, called the characteristic function $\Psi(\boldsymbol{x})$ (Ref. 24):

$$\Psi(\boldsymbol{x}) = s_f + P(\boldsymbol{x}) \tag{11}$$

The characteristic function $\Psi(\boldsymbol{x})$ is evaluated for each row of the inner array. The constraint violations are reflected in the characteristic function via the scale factor z. If z is too small, it may be difficult to obtain a feasible solution. However, a large z might ignore the robustness of the objective function represented as the standard deviation. Thus, an appropriate z has to be chosen deliberately. In the example problems, the scale factor is set to a value so that the order of the standard deviation is slightly higher than that of the penalty function. The scale factor z is imposed to emphasize the constraint violation.

B. Design Process

The overall process is the same as the approach for the unconstrained problem.[21] An optimum is evaluated by conventional optimization, which has a continuous design space. The number of levels is set to three for each design variable. The second level is fixed by the closest candidate from the continuous optimum. The first and third levels are fixed by the upper and lower ones around the second level.

An example is shown in Fig. 1. Suppose that the number of design variables is three and each design variable has five candidate values. When the closest values from the continuous optimum are $A_3B_2C_4$, the levels of design variables are selected as Fig. 1.

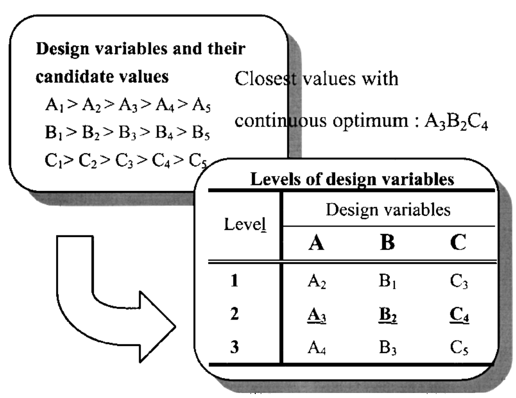

Fig. 1 Selection of level values.

The best condition can be selected from the full combinations of design variables. However, it is reasonable to select the smallest orthogonal array because full combinations are inefficient and costly. The smallest size of the inner array can be obtained by reflecting the number of the design variables. Also, when the interaction among the design variables is strong, the interaction should be considered when choosing the smallest size of an orthogonal array. An interaction occurs when simultaneously considered design variables have a different effect from the effect made by individual design variables.[25] However, it is not easy to grasp the strong interaction among the design variables in the structural design. Generally, the effect of interaction is ignored.

An appropriate orthogonal array can be selected to minimize the interaction effect as follows: For a problem with three design variables and three levels, the $L_9(3^4)$ orthogonal array of Table 1 is recommended so that two design variables are allotted for the first two columns, whereas the rest of the design variables are allotted for the fourth column. Then, the effects of C and $A \times B^2$ are confounded. For a design problem with more than three design variables and three levels, special orthogonal arrays such as $L_{18}(2^1 \times 3^7)$ and $L_{36}(2^{11} \times 3^{12})$ are strongly recommended. The $L_{18}(2^1 \times 3^7)$ orthogonal array in Table 2 has the advantage in that the effects of interactions are evenly distributed among the columns, with the exception of the relationship between columns 1 and 2. When a strong interaction exists, it should be considered in the selection process. However, it is very difficult because the interaction should be identified before the design process. In this research, the interaction effect is reduced by the proper choice of an orthogonal array as mentioned earlier.

The inner array is an orthogonal array that is used for a parameter design in the Taguchi method.[19] In a matrix experiment, experiments for a row of the inner array are conducted repeatedly to evaluate the standard deviation of the response. In numerical experiments, the exact response is calculated for given values. Therefore, an orthogonal array (outer array) is applied to a certain row to consider the tolerance of design variables. The design variables are perturbed according to the outer array. The tolerances of the design variables can be regarded as the variations of the design variables. The number of levels for the tolerances of a design variable is set to three because the nominal value, lower limit, and upper limit of a design variable are deliberated. The size of the outer arrays is chosen in the same way for the inner arrays.

The arrangement of the inner and the outer arrays is shown in Fig. 2 for a constrained problem. The number of experiments of the inner array is represented as N_{in} in Fig. 2. Each row of the inner array generates a value from the characteristic function given in Eq. (11), which is calculated from the outer array. The number of experiments of the outer array is N_{out}. N_{out} experiments are required to obtain the SNR or the standard deviation in unconstrained problems, whereas many experiments are required to obtain the characteristic function in constrained problems. In structural designs, an experiment means one finite element analysis.

After all of the characteristic functions from Ψ_1 to $\Psi_{N\text{in}}$ in Fig. 2 are calculated, the optimum level of each design variable is determined by the analysis of the characteristic function. The characteristic function is evaluated from $\sum \lambda_j$, z, $\max[0, v]$, and s_f for each row of an inner array.

Now the optimum levels are determined. The process is identical to the parameter design of the Taguchi method,[19] except that the characteristic function is used. As shown in Fig 2, suppose that we have N_{in} characteristic functions. The characteristic functions in the inner array are summed for each level of the design variables as shown in Table 3. The level with the smallest value is selected for the optimum level. In Table 3, l is the number of levels and $\sum \Psi_{ln}$ is the summation of the characteristic function with respect to the lth level of the nth design variable. The estimator of the characteristic function with respect to the optimum level is evaluated as follows[20]:

$$\hat{\Psi}(x) = m_{x1} + m_{x2} + \cdots + m_{xn} - (n-1)\bar{m} \tag{12}$$

where

$$\bar{m} = \frac{1}{N_{\text{in}}} \cdot \sum_{i=1}^{N_{\text{in}}} \Psi_i$$

Table 1 Orthogonal array, $L_9(3^4)$

	Column			
Experiment	1	2	3	4
1	0	0	0	0
2	0	1	1	1
3	0	2	2	2
4	1	0	1	2
5	1	1	2	0
6	1	2	0	1
7	2	0	2	1
8	2	1	0	2
9	2	2	1	0
DV	*A*	*B*		*C*

Table 2 Orthogonal array, $L_{18}(2^1 \times 3^7)$

	Column							
Experiment	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	0
2	0	0	1	1	1	1	1	1
3	0	0	2	2	2	2	2	2
4	0	1	0	0	1	1	2	2
5	0	1	1	1	2	2	0	0
6	0	1	2	2	0	0	1	1
7	0	2	0	1	0	2	1	2
8	0	2	1	2	1	0	2	0
9	0	2	2	0	2	1	0	1
10	1	0	0	2	2	1	1	0
11	1	0	1	0	0	2	2	1
12	1	0	2	1	1	0	0	2
13	1	1	0	1	2	0	2	1
14	1	1	1	2	0	1	0	2
15	1	1	2	0	1	2	1	0
16	1	2	0	2	1	2	0	1
17	1	2	1	0	2	0	1	2
18	1	2	2	1	0	1	2	0
DV		*A*	*B*	*C*	*D*	*E*	*F*	

Table 3 Analysis for $\Psi(x)$

	Design variable			
Level	x_1	x_2	.	x_n
1	$\sum\Psi_{11}$	$\sum\Psi_{12}$	.	$\sum\Psi_{1n}$
2	$\sum\Psi_{21}$	$\sum\Psi_{22}$	.	$\sum\Psi_{2n}$
.	.	.	.	
.	.	.	.	
l	$\sum\Psi_{l1}$	$\sum\Psi_{l2}$	.	$\sum\Psi_{ln}$

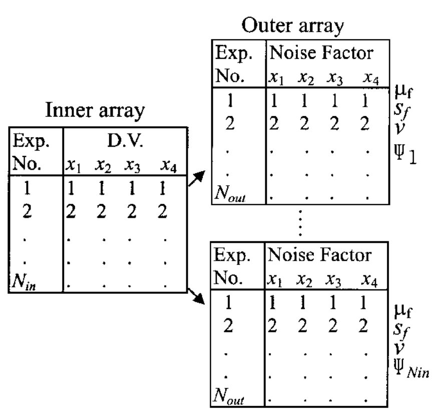

Fig. 2 Arrangement of inner and outer arrays for a constrained problem.

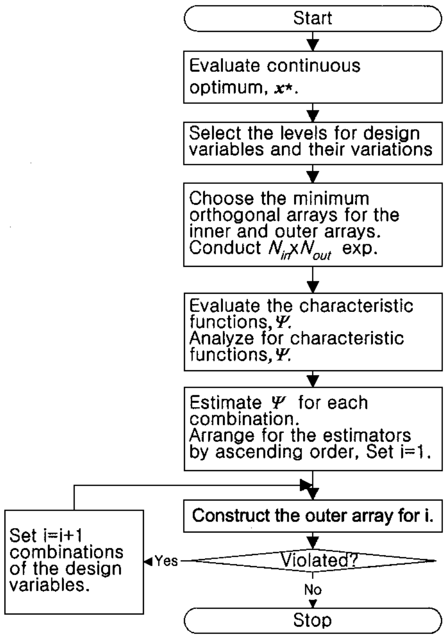

Fig. 3 Flowchart of discrete design for a constrained problem.

and m_{xn} is the summation of the characteristic function values to the optimum level of the design variable x_n divided by N_{in}/l.

However, it is not guaranteed that the optimum evaluated by the analysis of the characteristic function satisfies the imposed constraints. This is because the penalty function is an approximated function. Furthermore, the outer array does not include all of the combinations of the variations on the design variables. To increase the constraint feasibility, the scale factor can be made larger. However, an excessive scale factor leads to neglecting the effect of the standard deviation. Also, a large scale factor can induce an overdesign. The use of the scale factor gives the dimensional balance between the standard deviation and the penalty function. In this study, the scale factors are set to 0.1 and 1.0 so that the dimension of the standard deviation has the same or one higher order. The flow of the developed method is shown in Fig. 3.

The estimator evaluated by Eq. (12) has the smallest value. Because the optimum can violate some constraints, it is required that the combination with the next larger estimator be investigated. For an automatic loop, the estimators with respect to all combinations of the design variables are arranged by ascending order. Evaluating the estimators of all combinations is not an expensive process because the real function calculations are not required for determining the estimator. The ranking is decided by the ascending order of estimators. The combinations are evaluated according to the ranking until the feasibility condition is satisfied. An outer array is constructed for each combination. Then the penalty function is calculated, and the feasibility is checked. If the penalty function is zero, the constraints are satisfied in the range of variations. This process needs additional function calculation of the imposed constraints. The use of an outer array is cost effective compared to the full combinations of the variations on design variables. However, perfect feasibility is not guaranteed. This process is continued until the combination with the smallest estimator without violating any constraint in the outer array is discovered.

IV. Examples and Discussion

The designs of truss and beam structures are solved to illustrate the validity of the developed method. Examples consist of standard problems such as a three-bar truss and a two-member frame. As a practical example, it is applied to the design of a space frame in an electrical vehicle. For each problem, the standard deviation of the objective function and the constraint feasibility are evaluated for the robust optimum from the suggested method and the rounded-up values obtained from continuous optimization. The discrete values are selected around the optimum evaluated by conventional optimization using recursive quadratic programming.[22] The discrete values and variations are arbitrarily assigned.

Table 4 Levels of design variables (three-bar truss)

Array	Level	Design variable A_1	A_2	A_3
Inner ($\times 10^{-3}$ m^2)	1	5.806	1.935	4.516
	2	5.161	1.290	3.871
	3	4.516	0.645	3.226
Outer ($\times 10^{-5}$ m^2)	1	$A_1 + 6.452$	$A_2 + 6.452$	$A_3 + 6.452$
	2	A_1	A_2	A_3
	3	$A_1 - 6.452$	$A_2 - 6.452$	$A_3 - 6.452$

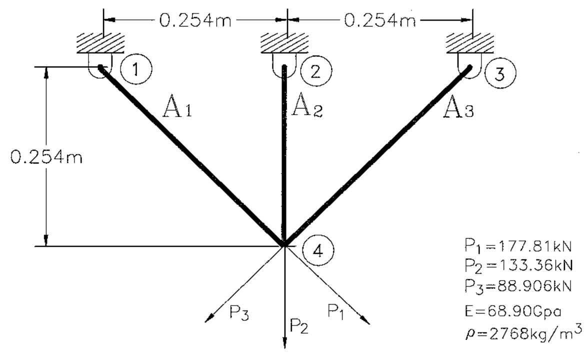

Fig. 4 Three-bar truss.

A. Three-Bar Truss

The design of the three-bar truss as shown in Fig. 4 is to determine the areas A_1, A_2, and A_3 (Ref. 2). The formulation for deterministic optimization is represented as

$$\begin{gathered}\text{minimize}\quad \text{weight}(A_1, A_2, A_3)\\ \text{subject to}\quad \sigma_i/\sigma_{\text{all}} - 1.0 \le 0.0\\ -F_i l_i^2 \big/ \pi^2 E A_i - 1.0 \le 0.0, \qquad u_4/u_{\text{all}} - 1.0 \le 0.0\\ v_4/v_{\text{all}} - 1.0 \le 0.0, \qquad i = 1, 2, 3\\ f_1/f_{\text{all}} - 1.0 \ge 0.0 \end{gathered} \tag{13}$$

Candidate values are

$$A_1(\times 10^{-3}\ \text{m}^2) = [3.871, 4.516, 5.161, 5.806, 6.451]$$

$$A_2(\times 10^{-3}\ \text{m}^2) = [0.645, 1.290, 1.935, 2.580, 3.225]$$

$$A_3(\times 10^{-3}\ \text{m}^2) = [2.581, 3.226, 3.871, 4.516, 5.161]$$

where σ_i, u_4, v_4, f_1, F_i, l_i, and E are the stress, the horizontal and the vertical displacements of node 4, the lowest frequency, the reaction force, the member length, and Young's modulus, respectively. The allowable stresses σ_{all} are 34.450 MPa for members 1 and 3 and 137.804 MPa for member 2. The allowable displacements u_{all} and v_{all} and the allowable frequency f_{all} are 0.127×10^{-3} m, 0.127×10^{-3} m, and 2500 Hz, respectively. The imposed 25 constraints and their orders are given in Ref. 1. The conventional optimum is evaluated as $\boldsymbol{x}^* = [5.346 \times 10^{-3}\ \text{m}^2,\ 7.719 \times 10^{-4}\ \text{m}^2,\ 3.713 \times 10^{-3}\ \text{m}^2]^T$.

Suppose that the first frequency should be retained without any drastic change. The standard deviation of Eq. (11) corresponds to the one with respect to the first frequency. With the candidate values of design variables and the continuous optimum, the levels of design variables for the inner array are determined as shown in Table 4. $L_9(3^4)$ orthogonal arrays are selected as the inner array and the outer arrays because the number of design variables is three. The variations of the design variables for the outer array are shown in Table 4. Table 5 gives the optimum combination of the design variables where the scale factor is set to 1.0.

In Table 5, the rank is arranged according to the estimator value. Each estimator is derived from Eq. (12). For the first rank, the combination is $[A_1^*, A_2^*, A_3^*]^T = [\text{level 2, level 1, level 1}]^T$. However,

Table 5 Optimum level for a constrained problem (three-bar truss)

SF, z	Rank	Estimator of Ψ ($\hat{\Psi}$)	Level A_1^*	A_2^*	A_3^*	s_f, Hz	Weight, kg	Constraint violated?
1.0	1	4.879	2	1	1	5.111	1.100	Yes
1.0	2	5.938	2	2	1	5.628	1.053	Yes
1.0	3	6.016	1	1	1	5.515	1.162	No
Rounded-up values			1	2	2	8.221	1.053	Yes

Table 6 Levels of design variables (one-bay, two-story frame)

Array	Level	Design variable w_1	h_1	t_1	w_2	h_2	t_2
Inner ($\times 10^{-3}$ m)	1	500	550	15	200	450	15
	2	450	500	13	150	400	13
	3	400	450	11	100	350	11
Outer ($\times 10^{-3}$ m)	1	$w_1 + 10$	$h_1 + 10$	$t_1 + 1$	$w_2 + 10$	$h_2 + 10$	$t_2 + 1$
	2	w_1	h_1	t_1	w_2	h_2	t_2
	3	$w_1 - 10$	$h_1 - 10$	$t_1 - 1$	$w_2 - 10$	$h_2 - 10$	$t_2 - 1$

Table 7 Optimum level for a constrained problem (one-bay, two-story frame)

SF, z	Rank	Estimator of Ψ ($\hat{\Psi}$)	Level w_1^*	h_1^*	t_1^*	w_2^*	h_2^*	t_2^*	s_w, kg	Weight, kg	Constraint violated?
1.0	1	27.15	2	1	1	2	1	2	265.3	5744.6	No
0.1	1	230.98	3	2	1	2	2	2	238.8	5171.1	Yes
0.1	2	231.22	3	3	1	2	2	2	226.6	4935.4	Yes
0.1	3	231.85	3	2	1	2	1	2	240.7	5273.2	Yes
0.1	4	232.09	3	3	1	2	1	2	228.6	5037.6	Yes
0.1	5	233.19	2	2	1	2	2	2	251.1	5406.8	No
Rounded-up values			2	2	3	1	2	3	254.2	4206.7	Yes

the optimum does not satisfy the imposed constraint of number 19 (Ref. 2) when the outer array of the combination is evaluated. The constraint for the second rank is also violated. Thus, the design with the third ranked combination is selected as the optimum design. The standard deviation for the robustness of the first frequency is slightly worse than that of the first ranked combination. On the contrary, the feasibility is enhanced. For the rounded-up values, the constraint is violated, and the standard deviation becomes worse than that of the third ranked combination of the robust optimum by 49%. As mentioned earlier, perfect feasibility is not guaranteed because it is investigated only by the combinations determined from the outer array.

B. One-Bay, Two-Story Frame

The design of the one-bay, two-story frame shown in Fig. 5 is to determine the width w, the height h, and the thickness t of each section under multiple loadings.[1] The conventional optimization is formulated as

$$\text{minimize} \quad \text{weight}(w_1, h_1, t_1, w_2, h_2, t_2)$$

$$\text{subject to} \quad \sigma_j/\sigma_{\text{all}} - 1.0 \le 0.0, \qquad j = 1, \ldots, 6$$

$$u_k/u_{\text{all}} - 1.0 \le 0.0, \qquad v_k/v_{\text{all}} - 1.0 \le 0.0, \qquad k = 2, \ldots, 5$$

$$f_1/f_{\text{all}} - 1.0 \ge 0.0 \tag{14}$$

Candidate values are

$$w_1, h_1, h_2(\times 10^{-3}\text{ m}) = [350, 400, 450, 500, 550]$$

$$w_2(\times 10^{-3}\text{ m}) = [100, 150, 200, 250, 300]$$

$$t_i(\times 10^{-3}\text{ m}) = [9, 11, 13, 15, 17], \qquad i = 1, 2$$

where $\sigma_{\text{all}} = 24.5$ MPa, $u_{\text{all}} = 8.0 \times 10^{-3}$ m, $v_{\text{all}} = 0.1 \times 10^{-3}$ m, and $f_{\text{all}} = 5.0$ Hz, respectively. The optimum in the continuous design space is

$$\boldsymbol{x}^* = \begin{bmatrix} w_1^* & h_1^* & t_1^* & w_2^* & h_2^* & t_2^* \end{bmatrix}^T$$
$$= [424.18 \times 10^{-3}\text{ m} \quad 484.79 \times 10^{-3}\text{ m} \quad 10.0 \times 10^{-3}\text{ m} \quad 152.39 \times 10^{-3}\text{ m} \quad 376.66 \times 10^{-3}\text{ m} \quad 10.0 \times 10^{-3}\text{ m}]^T$$

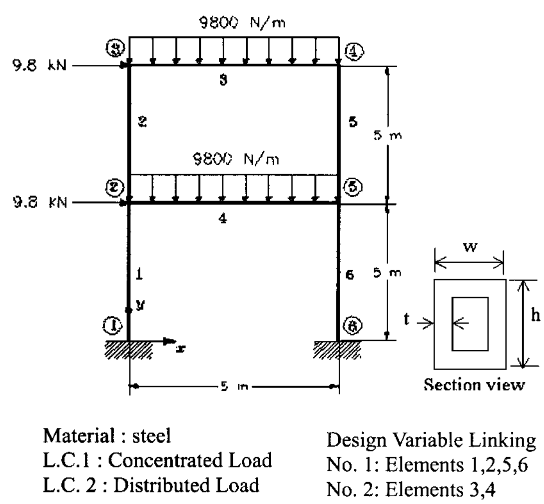

Fig. 5 One-bay, two-story frame.

The standard deviation of Eq. (11) is considered while the penalty function reflects all of the imposed constraints. Thus, the method provides the design with minimum variations of the weights while satisfying the constraints in the discrete design space. The discrete values around the conventional optimum and their variations are assumed as shown in Table 6. Because there are six design variables, $L_{18}(2^1 \times 3^7)$ orthogonal arrays are utilized as the inner and outer arrays. The first and the last columns in the arrays are empty. The optimum combinations with the scale factors of 1.0 and 0.1 are listed in Table 7. When the scale factor is 1.0, the combination with the smallest estimator satisfies the constraints. However, when the scale factor is set to 0.1, the optimum levels are selected as the design with the fifth rank. If the scale factor is large, the probability of satisfying the constraints is increased because the penalty function is emphasized more than the standard deviation in the characteristic function. That is, the robustness of the constraints is reflected more than that of the objective function. However, such a phenomenon leads to excessive design due to the increase of the objective function. For the rounded-up values, the constraint is violated, and the standard deviation becomes worse than that of the fifth-ranked combination of the robust optimum by 1%.

C. Space Frame of an Electrical Vehicle

The development of an electrical vehicle has been carried out to provide a pollution-free vehicle. The heavy structure deteriorates the overall performance of the vehicle. Thus, structural optimization is adopted to lighten the vehicle while maintaining mechanical performances.

As shown in Fig. 6, the aluminum space frame of an electrical vehicle consists of a roof, a support and rear frame, a floor panel, and strut parts.[11,26] The beam modeling for the space frame structures is more convenient than the monocoque type because of the simple sections of the body-in-white (BIW). In the conceptual or preliminary design stage, it is important to have good dynamic performances, which are represented as first torsion and first bending modes. In the initial design, the space frame has the first torsion mode of 32.01 Hz and the first bending mode of 38.94 Hz. These performances are utilized as constrained bounds in Eq. (15). Therefore, the formulation for deterministic optimization is represented as

$$\begin{aligned} &\text{minimize} \quad \text{weight}(w_1, h_1, t_1, \ldots, w_6, h_6, t_6) \\ &\text{subject to} \quad f_1(\text{first torsion}) \geq 32.01\ \text{Hz} \\ &f_2(\text{first bending}) \geq 38.94\ \text{Hz} \end{aligned} \tag{15}$$

Candidate values are

$$t_1(\times 10^{-3}\ \text{m}) = [4.8, 5.0, 5.2, 5.4, 5.6]$$

$$t_i(\times 10^{-3}\ \text{m}) = [2.8, 3.0, 3.2, 3.4, 3.6], \qquad i = 2, \ldots, 6$$

The design variables in the continuous design space are selected as widths, heights, and thicknesses of sections of the pillars and side members. The section views of the members are equal to those of the one-bay, two-story frame as shown in Fig. 5. The members with the design variables are shown as boldface lines in Fig. 6. The thicknesses of the frame should be selected from the standard products of discrete design variables. Thus, the thicknesses are used for design variables, and their discrete values around the continuous optimum are listed in Table 8.

Table 8 Levels of design variables (space frame)

Array	Level	Design variable t_1	t_2	t_3	t_4	t_5	t_6
Inner ($\times 10^{-3}$ m)	1	5.4	3.4	3.4	3.4	3.4	3.4
	2	5.2	3.2	3.2	3.2	3.2	3.2
	3	5.0	3.0	3.0	3.0	3.0	3.0
Outer ($i = 1, 6$)	1	$t_i + (t_i \times 0.1)$					
	2	t_i					
	3	$t_i - (t_i \times 0.1)$					

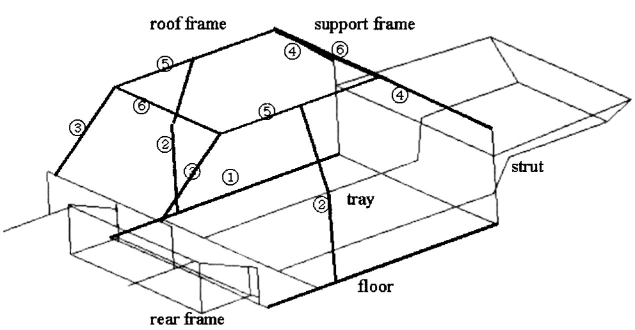

Fig. 6 Stick model of space frame in an electrical vehicle.

The tolerances of the design variables are set to have 10% of the design variables (DVs). In the same manner as with the one-bay, two-story frame, $L_{18}(2^1 \times 3^7)$ orthogonal arrays are utilized as the inner and outer arrays. In this design, Eq. (11) is composed into the standard deviation of BIW's weight and the penalty function of two dominant frequencies.

With the scale factor (SF) of 0.1, the optimum levels are determined as $[t_1, t_2, t_3, t_4, t_5, t_6]^T =$ [level 1, level 1, level 1, level 1, level 3, level 2]^T. This robust optimum has 32.12 Hz of the first frequency and 38.96 Hz of the second frequency. It is determined from the sixth ranked estimator for the characteristic functions. The standard deviations of the robust optimum and the rounded-up values are 0.112 and 0.105, respectively. It may seem that the rounded-up values have better robustness. However, the first constraint is violated, though its standard deviation is smaller than that of the robust optimum. Thus, the design determined from the rounded-up values is useless. This phenomenon is frequently found in the rounded-up design.

V. Conclusions

The following conclusions can be made from this study:

1) The discrete design is developed as a postprocess of constrained optimization in the continuous design space. It is used as a postprocess to increase the feasibility chance. However, the developed method can be used from the beginning without conventional optimization. Through the robust design procedure, a structure is designed to be less sensitive to the variations of DVs enhancing the feasibility of the constraint function. The constraint feasibility is investigated by ranking the estimators of the characteristic function. This method can be applied with low computing cost, whereas the existing discrete optimizations are excellent but expensive.

2) By definition of the characteristic function, the constrained optimization problems are solved by the parameter design of the Taguchi method.[19] The SNR of the Taguchi method is replaced by the characteristic function composed of the standard deviation and the penalty function. The standard deviation denotes the robustness of the object function. On the contrary, the penalty function represents the robustness of constraint functions. In the characteristic function, the reasonable SF prevents an excessive design and reduces the function calculations for investigating the feasibility of the constraints.

3) Orthogonal arrays from the experimental design are adopted successfully in the discrete design of structures. Most structural problems have interactions between the DVs, though their effects are not generally predicted. Thus, for the design problem with more than three DVs and three levels, it is strongly recommended that special orthogonal arrays such as $L_{18}(2^1 \times 3^7)$ and $L_{36}(2^{11} \times 3^{12})$ be selected so that the effects of interactions are evenly distributed among the columns. When a strong interaction exists, it should be considered in the design process. In future studies, the interaction of the structural characteristics must be investigated, and structural design must use the interaction.

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A. Messac
Associate Editor